Factoring trinomials

In general, we are factoring $ax^2 + bx + c$ where *a*, *b*, and *c* are real numbers.

To factor an expression means to write it as a product of factors instead of a sum of terms. The expression $3x^2 + 10x + 8$ can be thought of as the sum of three terms. The equivalent expression $(1x + 2)(3x + 4)$ can be thought of as the product of two factors.

This worksheet will work on our ability to factor trinomials, expressions with three terms like $3x^2 + 10x + 8$. To better understand how we factor this expression, we will start by looking at how two binomials (expressions with two terms) multiply together. Usually, the product of two binomials is a trinomial. Factoring is simply going the opposite way.

Let's say we multiply the following two binomials. I will write the steps of **FOIL** out explicitly. In particular, this will help us make sense of the first factoring method on this worksheet, the A-C method. I labeled the lines so I can refer to them later.

$(1x + 2)(3x + 4)$	line 1
$= 3x^2 + 4x + 6x + 8$	line 2
F O I L	
$= 3x^2 + 10x + 8$	line 3

The A-C method:

Notice the **O** and the **I** terms add together to make the 10x term.

Remember this expression $3x^2 + 10x + 8$ is in the general form $ax^2 + bx + c$. It is this *a* and *c* to which the A-C method refers. We will use this specific example and others to make sense of the A-C method.

This is the key to the whole thing. Notice, on line 3 above, how the 3 (of $3x^2$) and the 8 (the constant term) multiply to make 24. And also, on line 2, how the 4 and the 6 (of $4x + 6x$) multiply to make 24.

Another thing to keep in mind is how the $4x + 6x$ (line 2) added to make the $10x$ in line 3.

Let's look at a few more examples. Find each product by using FOIL. Write out all four terms of FOIL and then add them as I did above. Yes, I want you to write it out explicitly.

a.) $(x+2)(x+3)$

b.) $(2x+4)(x+3)$

c.) $(2x-3)(x-3)$

Notice in each case, the product of the **F** and **L** coefficients equals the product of the **O** and the **I** coefficients.

What we have been doing is turning the **product** of two numbers into a **sum** of three terms. When we factor, we go the other way. We turn a sum of terms into a product of factors. Suppose we are given $3x^2+4x-4$ and are asked to factor it. We want to write it as "something times something" instead of the sum it is now.

The A-C method will tell us to multiply *a* and *c* of our trinomial (ax^2+bx+c), and then find two numbers whose product is the same but also add to *b*. We will use this knowledge to rewrite the *bx* term in the trinomial. We will then factor by grouping which is described on the next page.

We will work with factoring $3x^2+4x-4$ as an example of the A-C method.
First, let's go over some preliminary information that we need for the A-C method of factoring. It's called "factoring by grouping" and it allows us to factor expressions with four terms such as $8x^2+4x-6x-3$. (This only works if it is indeed factorable. Many expressions are not.)

Factoring by Grouping:
Factoring by grouping means just that. We group the first two terms and factor something out of them. Then we group the last two terms and factor something out of them. If we do it right, what is left over from the first two terms will be the same as what's left over from the last two terms. Here is an example.

Factor $4x$ from $8x^2 + 4x$

$$8x^2 + 4x - 6x - 3$$

Factor –3 from – 6x – 3

$$= 4x\underline{\underline{(2x+1)}} - 3\underline{\underline{(2x+1)}}$$

$$= (4x-3)(2x+1)$$

Notice the common factor of $2x + 1$ is left over. I've underlined it to show it off. Factor it out.

Explaining what was done:
Notice how we started with four terms. From the first two terms, we factored out $4x$. From the last two terms, we factored out -3.

This gave us two terms (they are $4x(2x + 1)$ and $3(2x + 1)$) that have a common factor of $2x + 1$. We then factored the $2x + 1$ out and got $(4x – 3)(2x + 1)$.

Notice this results in the factored form of $8x^2 + 4x - 6x - 3$ or $(4x-3)(2x+1)$.

Practice by factoring the following expression by grouping.
$2x^2 - 6x - 3x + 9$

Check your answer by looking at problem *c* on page 2.

Let’s try factoring $3x^2 + 4x - 4$. We need two numbers that multiply to make 3(-4) or –12 and add to make 4. Think of the possible factors of –12.

Possible factors of -12		
-1 and 12	-2 and 6	-3 and 4
-12 and 1	-6 and 2	-4 and 3

There is only one pair that adds to 4. So we’ll write

$$\begin{aligned} &3x^2 + 4x - 4 \\ &= 3x^2 - 2x + 6x - 4 \end{aligned}$$

Now we’ll start factoring by grouping. We’ll pull out a common factor from the first two terms (x) and a common factor from the last two terms (2). So we have

$$\begin{aligned} &3x^2 - 2x + 6x - 4 \\ &= x(3x-2) + 2(3x-2) \\ &= (x+2)(3x-2) \end{aligned}$$

Notice this leaves two terms in the middle line, each with a factor of $3x – 2$. We factor this $3x – 2$ out and get $(x+2)(3x-2)$. This writes the sum we started with as a product of two numbers, $x + 2$ and $3x – 2$.

Try this guided example on your own.
Factor $3x^2 - 11x + 10$. Start by finding the possible factors of 3(10) or 30. (Do not forget the negatives.)

Possible factors of 30			

Now choose the pair that adds to –11 and rewrite the trinomial with four terms.

Now factor by grouping and finish with the factored form of $3x^2 - 11x + 10$. Circle this final answer. You may want to FOIL it out in your head or on scratch paper to check your answer.

Use the A-C method to factor the following.

a.) $10x^2 + 4x - 6$

b.) $12x^2 - 7x - 10$

c.) $2x^2 + 10x + 12$

d.) $150x^2 - 150x - 36$

Cross-product method:

This is essentially a way to write the information we need in an organized way. Let's factor $3x^2 + 4x - 4$ again. We write two factors of $3x^2$, in a column. Then we write the factors of -4 in a second column beside the first.

$$\begin{array}{lcr} 3x & & -4 \\ x & & 1 \end{array}$$

We then multiply as the arrows indicate. We get $3x(1)$ and $x(-4)$ or $3x$ and $-4x$. We add these; if they add to $4x$ (our middle term in $3x^2 + 4x - 4$) we are done. But this is not the case ($3x + -4x = -x$) so we go on. Try two more factors of -4 in the second column.

$$\begin{array}{lcr} 3x & & -2 \\ x & & 2 \end{array}$$

Notice this time we get $3x(2)$ and $x(-2)$ or $6x$ and $-2x$, which add to $4x$. All we need to do now is write the factors. Going across the top row, we get $3x + -2$ or $3x - 2$. Going across the bottom row, we get $x + 2$. So, the factored form of $3x^2 + 4x - 4$ is $(3x - 2)(x + 2)$.

Use the Cross-product method to factor the following.

a.) $6x^2 + x - 12$ (Hint: Remember there are essentially two ways to factor $6x^2$. They are $6x$ and x or $3x$ and $2x$. You'll have to try them both to see which works.)

b.) $5x^2 + 18x - 8$

Reverse FOIL method:

This method is also a way to write the information in an organized fashion. I call it Reverse FOIL because it helps to understand how FOIL works when multiplying two binomials. (A binomial is a polynomial with two terms like "x + 4".) Consider the multiplication problem below. Factoring goes the opposite way.

$(x+4)(2x-3)=2x^2-3x+8x-12=2x^2+5x-12$

As the example illustrates, FOIL involves multiplying the First, Outside, Inside, and Last terms of the two binomials. The First part ($2x^2$) is the product of x and $2x$. The Outside part ($-3x$) is the product of x and -3. The Inside part (8x) is the product of 4 and $2x$. The Last part (-12) is the product of 4 and -3. Notice how the Inside and Outside parts add to make the $5x$ in the final answer.

Now that we've seen how FOIL works, let's go the opposite way. Let's start with $2x^2+5x-12$ and see if we can factor it. We'll start off by writing the two sets of parentheses that we know must be a part of it.

$2x^2+5x-12=(\qquad)(\qquad)$

Then we need to think about the term $2x^2$. Again, recall this term would be formed by multiplying the First terms in the two binomials. So let's try $2x$ and x for these terms. So we write it in.

$2x^2+5x-12=(2x\qquad)(x\qquad)$

Now, we need two numbers that multiply to make -12. These will be the Last terms in our answer. Factors of -12 are listed below.

Possible factors of -12		
-1 and 12	-2 and 6	-3 and 4
-12 and 1	-6 and 2	-4 and 3

You can simply put each pair into the parentheses and check it to see if the pair works. Once you find one pair that works, you can stop. So try $(2x-1)(x+12)$ but that doesn't multiply to make $2x^2+5x-12$, so that's not right. Try $(2x-2)(x+6)$ and so on. You will find that only $(2x-3)(x+4)$ works.

If you want to do less trial and error, you can think about it the following way. To make it easier to discuss, let's write our incomplete factorization as $2x^2+5x-12=(2x+A)(x+B)$. Recall how the term $5x$ must be formed by the sum of the Outside and Inside terms on the right. This means that $5x$ must be the sum of Ax and 2Bx. In other words "one of the factors plus twice the other should equal 5". We see the only pair of factors that's true of is -3 and 4.

Try a guided example.

Factor $3x^2 + 11x - 20$. First, fill in the First terms in the parentheses. They should multiply to make $3x^2$ and both contain an *x*.

$$3x^2 + 11x - 20 = (\qquad\qquad)(\qquad\qquad)$$

Now write down the factors of -20.

Possible factors of -20		

Try each pair of factors in the parentheses to see which one works. Some parentheses are provided below. Stop when you find the pair that works.

$$3x^2 + 11x - 20 = (3x \qquad\quad)(x \qquad\quad) \text{ ????}$$
$$3x^2 + 11x - 20 = (3x \qquad\quad)(x \qquad\quad) \text{ ????}$$
$$3x^2 + 11x - 20 = (3x \qquad\quad)(x \qquad\quad) \text{ ????}$$

Use the Reverse FOIL method to factor these expressions.

a.) $2x^2 - 9x - 18$

b.) $4x^2 - 4x - 15$ (This is more complicated. The parentheses could be written as $(4x \qquad)(x \qquad)$ or $(2x \qquad)(2x \qquad)$. You'll have to try them both and see which eventually works.)