Rational Functions: Vertical asymptotes NAME:

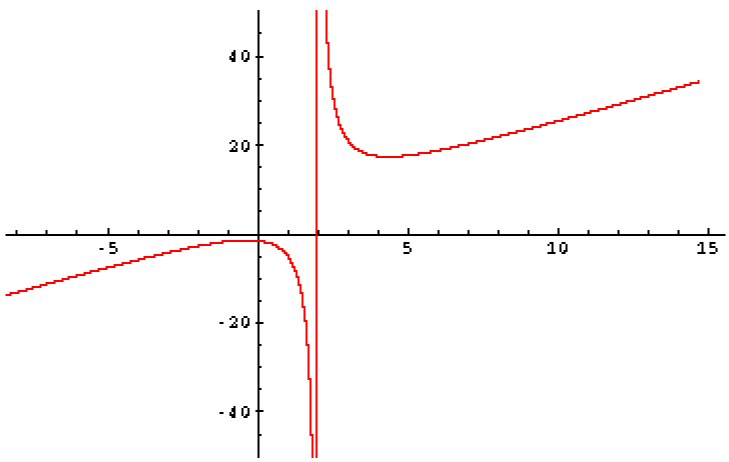
This worksheet is designed to help you understand why and where vertical asymptotes exist on the graphs of rational functions.

Recall a rational function is a function that can be written as a fraction whose numerator (top) and denominator (bottom) are both polynomial functions. You should recognize all the functions on this worksheet are indeed rational functions.

Remember a fraction is said to be "undefined" if the denominator is zero.

1. Let's start with the function $g(x) = \dfrac{2x^2+3}{x-2}$. What is $g(2)$?

2. Using your grapher, verify that the graph of $g(x) = \dfrac{2x^2+3}{x-2}$ is shown below. Notice the vertical line at x = 2. This is called a **vertical asymptote**. Vertical asymptotes occur at x values where the rational function is undefined.

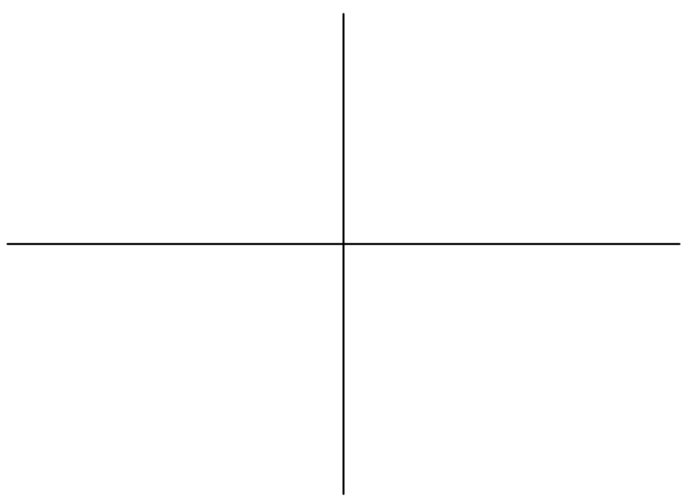


3. Where do you think the vertical asymptotes would occur on the graph of

$f(x) = \dfrac{4x+1}{(x+1)(x-3)}$?

4. Graph $f(x) = \dfrac{4x+1}{(x+1)(x-3)}$ to see if you were correct. Mark and label tick marks on the x-axis to make your graph more accurate. Draw the vertical asymptotes as dashed lines.

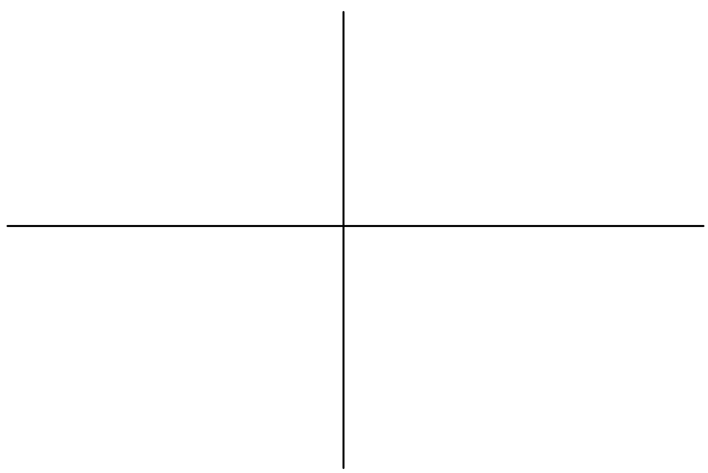
(Note: When you enter this into your calculator, you need to make sure you have parentheses around the entire top and entire bottom; it should look like $y_1 = (4x+1)/((x+1)(x-3))$. Notice the second set of parentheses on the bottom.)



5. Where do you think the vertical asymptotes would occur on the graph of

$f(x) = \dfrac{x-4}{x^2+x-6}$? (HINT: Solve $x^2 + x - 6 = 0$ to see where the denominator is zero.)

6. Graph $f(x) = \dfrac{x-4}{x^2+x-6}$ to see if you were correct. Mark and label tick marks on the *x*-axis to make your graph more accurate. (You might need to zoom in to see the part of the graph that is right of 2. I used the ZOOMIN feature.) Draw the vertical asymptotes as dashed lines.

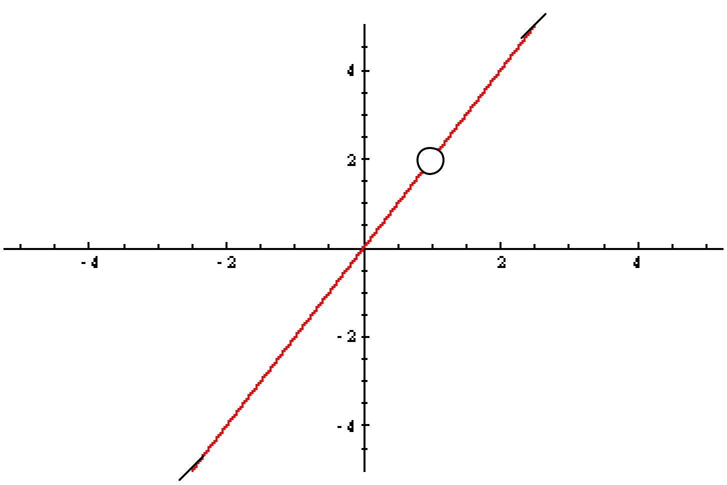


7. Where do you think the vertical asymptotes would occur on the graph of

$f(x) = \dfrac{2x^2 - 2x}{x-1}$?

8. Graph $f(x)=\dfrac{2x^2-2x}{x-1}$ to see if you were correct. You will notice there is not a vertical asymptote at $x = 1$. Why do you think that is? Simplify $\dfrac{2x^2-2x}{x-1}$ to find out what's going on.

9. Notice $\dfrac{2x^2-2x}{x-1}=\dfrac{2x(x-1)}{(x-1)}=2x$. This means the function $f(x)=\dfrac{2x^2-2x}{x-1}$ is equivalent to the function $y=2x$ except where x is 1. So the graphs are identical except where x is 1. When x is 1, the graph of $f(x)=\dfrac{2x^2-2x}{x-1}$ has a hole. The graph of *f(x)* really looks like the following.



The main point of this worksheet is that vertical asymptotes of rational functions are found at the x values that make the bottom zero (where the rational function is undefined). The exception to this occurs when there are common factors on the top and bottom (like $(x – 1)$ in number 8.) In these instances, there is a hole at the x values that make the bottom zero.