Logarithm Rules Worksheet Solutions NAME:

Remember to think of $\log_b x$ as the number to which I raise b to get x. This is very important in the study of logs.

Let b, v, and w be positive real numbers where b is not equal to one. Let k be a real number.

1. In words, what is $\log_b b$? It's the number to which I raise __*b*___ to get __*b*___.

What does this number we call $\log_b b$ have to be?

Any number raised to 1 is itself. The number you raise b to, to get itself, has to be 1. So $\log_b b = 1$.

2. In words, what is $\log_b 1$? It's the number to which I raise __*b*___ to get __*1*___.

What does this number we call $\log_b 1$ have to be?

Any number raised to 0 is 1. The number you raise b to, to get 1, has to be 0. So $\log_b 1 = 0$.

3. In words, what is $\log_b(b^k)$? It's the number to which I raise __*b*___ to get __b^k___.

What does this number we call $\log_b(b^k)$ have to be?

This is sort of a trick question. What do I raise 2 to, to get 2^3? (Well, 3, silly. Don't ask stupid questions.) So, what do we raise b to, to get b^k? (Well, k, silly. I told you not to ask stupid questions.) So $\log_b(b^k) = k$.

4. Now $\log_b v$ is the number to which I raise b to get v. Therefore b raised to this power or $b^{\log_b v}$ should be what number?

We have to hold in our head that $\log_b v$ *is the number to which I raise b to get v. This is crucial. See this "* $\log_b v$ *" as one entity. It's the number I raise b to, to get v. Now, when we write* $b^{\log_b v}$ *, we have taken b and raised it this number. I should get v. So* $b^{\log_b v} = v$. *We can also see this be looking at specific examples. We see that* $10^{\log_{10} 1000} = 10^3 = 1000$.

5. Complete the table.

$\log_3 3 =$ *1*	$\log_3 9 =$ *2*	$\log_3 27 =$ *3*
$\log_5 25 =$ *2*	$\log_5 5 =$ *1*	$\log_5 125 =$ *3*
$\log_2 16 =$ *4*	$\log_2 4 =$ *2*	$\log_2 64 =$ *6*

Now use the three rows of the table to determine the relationship between $\log_b v$, $\log_b w$, and $\log_b (vw)$. Write the rule down and then show it works using an example from the table.

In the first row of the table, notice how we have the log of 3, the log of 9, and the log of their product 27. The second row finds the log of 25, the log of 5, and the log of their product 125. This is true of the third row too. What is the connection between (first row answers) 1 and 2, and 3? What is the connection between (third row answers) 4 and 2, and 6? We notice if we add the first two columns, we get the third column. So we see the rule emerge: $\log_b(v) + \log_b(w) = \log_b(vw)$. *As an example,* $\log_2 16 + \log_2 4 = \log_2 64$. *This is one of the many rules we will use to manipulate logs.*

6. Complete the table.

$\log_3 27 =$ *3*	$\log_3 9 =$ *2*	$\log_3 3 =$ *1*
$\log_5 125 =$ *3*	$\log_5 25 =$ *2*	$\log_5 5 =$ *1*
$\log_2 64 =$ *6*	$\log_2 4 =$ *2*	$\log_2 16 =$ *4*

Now use the three rows of the table to determine the relationship between $\log_b v$, $\log_b w$, and $\log_b (\frac{v}{w})$. Write the rule down and then show it works using an example from the table.

The reasoning is similar to that used above. The rule we are after is $\log_b(v) - \log_b(w) = \log_b(\frac{v}{w})$. *As an example,* $\log_2 64 - \log_2 4 = \log_2 16$.

7. Complete the table. You may use your calculator for the first two. Round your answers to three decimal places. The last one you should be able to work out logically.

$\log(3^5) =$ *2.386*	$5 * \log 3 =$ *2.386*
$\log_e(6^2) =$ *3.584*	$2 * \log_e 6 =$ *3.584*
$\log_5(5^2) =$ *2*	$2 * \log_5 5 =$ *2*

Now use the three rows of the table to determine the relationship between $\log_b v^k$ and $k * \log_b v$. Write the rule down and then show it works using an example from the table.

To find $\log_5(5^2)$ *by hand, use the rule developed in #3. To find* $2 * \log_5 5$, *find* $\log_5 5$ *and then multiply by 2. You should see that, for each example above,* $\log_b v^k = k * \log_b v$. *As an example,* $\log_5(5^2) = 2 * \log_5 5$.

Additional note concerning rules $\log_b(b^k) = k$ *and* $b^{\log_b v} = v$ *:*

These two rules are true because of the inverse relationship between the exponential function $f(x) = b^x$ *and the logarithmic function* $g(x) = \log_b(x)$. *Recall inverse functions undo each other. If we apply the exponential function to a number k, we get* b^k. *Then apply the logarithmic function to this number, getting us* $\log_b(b^k)$. *But since the two functions are inverses and undo each other, we should end up with k again. That is why* $\log_b(b^k)$ *must equal k.*

To better understand, let's use real numbers. Consider the inverse functions $f(x) = 2^x$ *and* $g(x) = \log_2(x)$. *Apply the rule of f to the number 3. This gets us* $f(3) = 2^3 = 8$. *Then apply the inverse to this number 8. This gets us* $g(8) = \log_2(8) = 3$. *We ended up with the 3 we started with. The logarithmic function undid the exponential function. We could have written this as* $\log_2(2^3) = 3$.

Going the other way, apply the rule of g to the number 8. This gets us $g(8) = \log_2(8) = 3$. *Then apply the rule of f to this answer. This gets us* $f(3) = 2^3 = 8$. *We are back at the 8 we started with. The exponential function undid what the logarithmic function did. We could have written this as* $2^{\log_2(8)} = 8$.