Transformations NAME:

## Part I:

Graph carefully. Check your values with your neighbors often to make sure you are on the right track.

1. Complete the table and graph $f(x)=x^{2}$ and $h(x)=x^{2}+3$.

| x | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $f(x)=x^{2}$ |  |  |  |  |  |  |  |
| $h(x)=x^{2}+3$ |  |  |  |  |  |  |  |



Notice the $y$ values of $h(x)=x^{2}+3$ are simply the $y$ values of $f(x)=x^{2}$ with 3 added. Since we are adding 3 to each $y$ value, we see the graph of $f(x)=x^{2}$ is shifted upward by 3 units to form the graph of $h(x)=x^{2}+3$. Did this happen in your graphs? This is called a vertical shift (up).
2. Complete the table and graph $f(x)=x^{2}$ and $g(x)=x^{2}-2$.

| X | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f(x)=x^{2}$ |  |  |  |  |  |  |  |
| $g(x)=x^{2}-2$ |  |  |  |  |  |  |  |



Notice the $y$ values of $g(x)=x^{2}-2$ are simply the $y$ values of $f(x)=x^{2}$ with 2 subtracted. Since we are subtracting 2 from each $y$ value, we see the graph of $f(x)=x^{2}$ is shifted two units downward to form the graph of $g(x)=x^{2}-2$. Did this happen in your graphs? This is called a vertical shift (down).
3. Complete the table and graph $f(x)=x^{2}$ and $r(x)=(x+3)^{2}$.

| x | -9 | -6 | -3 | 0 | 3 | 6 | 9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $f(x)=x^{2}$ |  |  |  |  |  |  |  |
| $r(x)=(x+3)^{2}$ |  |  |  |  |  |  |  |



This is a little harder to see. But do you notice how the $y$ values of $r(x)=(x+3)^{2}$ are simply the $y$ values of $f(x)=x^{2}$ but shifted to the left 3 units. [Notice $r(-3)=f(0)$ and $r(6)=f(9)$.] Since we are adding 3 to the $x$ values before we square it, we see the graph of $f(x)=x^{2}$ is shifted to the left 3 units to form the graph of $r(x)=(x+3)^{2}$. Did this happen in your graphs? This is called a horizontal shift (to the left).
4. Complete the table and graph $f(x)=x^{2}$ and $k(x)=(x-3)^{2}$.

| x | -9 | -6 | -3 | 0 | 3 | 6 | 9 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f(x)=x^{2}$ |  |  |  |  |  |  |  |
| $k(x)=(x-3)^{2}$ |  |  |  |  |  |  |  |



Notice how the $y$ values of $k(x)=(x-3)^{2}$ are simply the $y$ values of $f(x)=x^{2}$ but shifted to the right 3 units. [Notice how $k(-3)=f(-6)$ and $k(6)=f(3)$.] Since we are subtracting 3 from each $x$ value before we square it, we see the graph of $f(x)=x^{2}$ is shifted 3 units to the right to form the graph of $k(x)=(x-3)^{2}$. Did this happen to your graphs? This is called a horizontal shift (to the right).
5. Complete the table and graph $f(x)=x^{2}$ and $d(x)=2 x^{2}$.

| x | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f(x)=x^{2}$ |  |  |  |  |  |  |  |
| $d(x)=2 x^{2}$ |  |  |  |  |  |  |  |



Notice the $y$ values of $d(x)=2 x^{2}$ are simply the $y$ values of $f(x)=x^{2}$ multiplied by 2 .
Since we are doubling each $y$ value, we see the graph of $f(x)=x^{2}$ is stretched vertically by a factor of 2 . Did this happen to your graphs? This is called a vertical stretch.
6. Complete the table and graph $f(x)=x^{2}$ and $d(x)=\frac{1}{3} x^{2}$.

| x | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $f(x)=x^{2}$ |  |  |  |  |  |  |  |
| $d(x)=\frac{1}{3} x^{2}$ |  |  |  |  |  |  |  |



Notice the $y$ values of $d(x)=\frac{1}{3} x^{2}$ are simply the $y$ values of $f(x)=x^{2}$ multiplied by $\frac{1}{3}$.
Since we are dividing each $y$ value by 3 , we see the graph of $f(x)=x^{2}$ is compressed (squashed down) vertically by a factor of $\frac{1}{3}$ to form the graph of $d(x)=\frac{1}{3} x^{2}$. Did this happen to your graphs? This is called a vertical compression.
7. Complete the table and graph $f(x)=x^{2}$ and $h(x)=-x^{2}$.

| x | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f(x)=x^{2}$ |  |  |  |  |  |  |  |
| $h(x)=-x^{2}$ |  |  |  |  |  |  |  |



Notice the $y$ values of $h(x)=-x^{2}$ are simply the negatives of the $y$ values of $f(x)=x^{2}$. Since we are making each $y$ value negative, the graph of $f(x)=x^{2}$ is flipped over the $x$ axis to form the graph of $h(x)=-x^{2}$. Did this happen to your graph? This is called a reflection about the $x$-axis.
8. Complete the table and graph $f(x)=x^{3}$ and $h(x)=(-x)^{3}$. [Be careful here. Notice $h(3)=(-3)^{3}=-27$ and $h(-3)=(--3)^{3}=3^{3}=27$.]

| x | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $f(x)=x^{3}$ |  |  |  |  |  |  |  |
| $h(x)=(-x)^{3}$ |  |  |  |  |  |  |  |



Notice the $y$ values of $h(x)=(-x)^{3}$ are simply the $y$ values of $f(x)=x^{3}$ but in reverse order. Since we are turning each $x$ value into its negative before we cube $i t$, the graph of $f(x)=x^{3}$ is flipped over the $y$-axis to form the graph of $h(x)=(-x)^{3}$. Did this happen to your graphs? This is called a reflection about the $\boldsymbol{y}$-axis.

## Part II:

In general, what happens to the graph of $f(x)$ when the following transformations are performed on it? Then, for future reference, denote the question number from above that serves as an example for each transformation. The first one is done for you.

| Let c be a positive real <br> number. | Transformation of <br> graph | Number of question which <br> is an example of the <br> transformation |
| :---: | :---: | :---: |
| $c^{*} f(x)$, where $\mathrm{c}>1$ | Vertical stretch by a <br> factor of c |  |
| $c^{*} f(x)$, where $0<\mathrm{c}<1$ |  | 5 |
| $f(x-c)$ |  |  |
| $f(x+c)$ |  |  |
| $f(x)+\mathrm{c}$ |  |  |
| $f(x)-\mathrm{c}$ |  |  |
| $-f(x)$ |  |  |
| $f(-x)$ |  |  |

