Working with Percents

Percent means “parts per hundred” or “for every hundred”

Can write as $\frac{40}{100}$ or .40 or 40% - fractions or decimals or percents

Converting and rewriting decimals, percents and fractions:

To **convert** a decimal number to a percent, multiply by 100% or move the decimal point two digits to the right

0.70 = 0.70 x 100% = 70%
0.70 becomes 70.% or 70%

To **rewrite** a fraction as a percent, either divide the numerator by the denominator and then multiply by 100%:

$$\frac{5}{16} = 0.3125 = 0.3125 \times 100\% = 31.25\%$$

OR

rework it to make the denominator 100 and reduce common factors:

$$\frac{3}{20} = \frac{3 \times 5}{20 \times 5} = \frac{15}{100} = 15\%$$

To change a percent to a fraction, divide by 100% and reduce to lowest terms:

$$36\% = \frac{36}{100} = \frac{9 \times 4}{25 \times 4} = \frac{9}{25}$$

3 Basic Percent Problems

For each type of problem, you’ll work with the following elements. You’ll have to find one of these when the other two are known.

Base or **Total (T)** amount or standard used for a comparison
Percentage or **Part (P)** being compared with the base or total
Percent or **Rate (%)** that indicates the relationship of the percentage to the base (i.e., the part to the total)

Follow these steps:

1. Translate the problem sentence into a math statement. For example, 30% of what number is 16:

30% x ☐ = 16 You can use *x* or ? or ☐ for the unknown.

2. Label which numbers are the base or total (T), the percent (%), and the percentage or part (P):

30% x ☐ = 16

↑ ↑ ↑

% T P

3. Rearrange the equation so that the unknown quantity is alone on the left of the equal sign and the other quantities are on the right of the equal sign.

$$\square = 16 \div 30\% = \frac{16}{30\%} = \frac{16}{.30}$$

4. Make a reasonable estimate of the answer. Guess, but guess carefully. You know, for example, that 30% is a little less than one third and you could compute that one third of 50 is about 17, so you can guess the neighborhood the number is in.

5. Solve the problem by doing the arithmetic. You must rewrite percents as fractions or decimals before you can use them in multiplication or division.

16 ÷ .30 = 53.33 (round to *x* decimal places, usually two)

6. Check your answer against the original guess. Are they close? If not, you've probably made a mistake and should repeat your work.

7. Double-check by putting the answer number you have found back into the original problem or equation to see it if makes sense.

30% of 53.33 = ??

<u>Type 1 Problem:</u>
Stated like this: "Find 30% of 50" or "What is 30% of 50?" or "30% of 50 is what number?"

Work through the steps.
Find the answer.
Hint: This is the easiest one to do with your calculator: just multiply 50 times .3 (30% changed to decimals) on your calculator.
Answer: 15

Type 2 Problem:
These problems require that you find the rate or percent. They are stated like "7 is what percent of 16?" or "Find what percent 7 is of 16" or "What percent of 16 is 7?"

☐% x 16 = 7
% x T = P

$$\square\% = \frac{7}{16}$$

The guess: $\frac{7}{16}$ is very close to $\frac{8}{16}$ or $\frac{1}{2}$ or 50%, so the answer will be a little less than 50%.

Answer: 43.75%

Type 3 Problem:
Requires that you find the total given the percent and the percentage or part. Usually stated as "8.7 is 30% of what number?" or "Find a number such that 30% of it is 8.7" or "30% of what number is equal to 8.7?"

30% x ☐ = 8.7
% x T = P

$$\square = \frac{8.7}{30\%}$$

Answer: 29

The IS-OF Method of solving percent problems:

- The IS number is always found next to the word *is*, or *equals*, or *is equal to*
- The OF number is always found next to the word *of* in the problem
- If a problem is obviously a Type 1 problem, the numbers should be multiplied ("What is 30% of 50?")

- Otherwise, divide the IS number by the OF number to find the answer ("What percent of 16 is 7?")

$\frac{\text{IS number}}{\text{OF number}} = \frac{7}{16} = \frac{700\%}{16} = 43\ ¾\%$ or 43.75%

Several Uses of Percentages

- Figure tips
  The director and several managers of the Big River County Library go to lunch. The bill is $38.75. They would like to leave a 15% tip. What would that be?

  $38.75 x .15 = $5.81

  If all five managers agreed to split the bill plus tip evenly, how much would each owe?

  $38.75 + $5.81 = $44.56 ÷ 5 = $8.92 (I rounded up)

- Figure discounts
  Baker & Taylor has negotiated a 40% discount on materials for the Big River County Library. Assuming all material types will receive this discount, what is the buying power of $9,000?

  40% of $9,000 is $3600. You could add that to the original $9,000 for a total buying power of $12,600—or you could figure 140% of $9,000, which is also $12,600.

- Calculate increases or decreases. There are two steps involved:

  - Step 1: find the *amount* of the increase or decrease.
    Last year a new globe cost $185. It now costs $215.
    $215 - $185 = $30

  - Step 2: find the percent change as a part of the original amount.
    Percent increase or decrease = $\frac{\text{amount of increase or decrease}}{\text{original amount}}$
    $\frac{\$30}{\$185}$ = 16% increase in cost

Common problems to avoid

- Base numbers so small that the change relationships are meaningless
- Not identifying the proper original number or amount. Double check or practice on numbers where you can easily see the relationship. Think about how you're trying to make your argument or present your data.

- Determine whether it will be more effective to talk in terms of percentages or in terms of the numbers themselves. Be sure you know what the base numbers are that others are using.
- Understand the difference between percentages, percentage points, and the numbers they relate to:

  Last year I spent 18% of my materials budget on AV materials. This year I plan on spending 20%. What is the difference?

  Too many people say that the difference is 2%. In reality, the difference is 11%. The dollar difference and absolute difference will depend, in addition, on the dollar amounts being talked about. In other words, 18% of x may be more than 20% of y.

- A percentage increase followed by a percentage decrease does not leave you back where you started. It leaves you worse off. If you take a 10% cut for each of two years, then get a 20% increase, you WILL NOT be back where you started.

  Example: $100,000 cut 10% = $90,000
  $ 90,000 cut 10% = $81,000
  $ 81,000 increased 20% = $97,200
  What percentage have you lost?

  You receive a 5% performance bonus on your salary of $50,000 for one year. That brings your salary plus bonus to $52,500. At the end of the year, your salary is reduced by 5% since the bonus was only for one year. You are dismayed to see that you are now making $49,875 a year. What happened?

- Doubling, tripling, etc. can be tricky. When you double a number, the percentage increase is 100%. When you triple a number, it's 200. Use the formula we learned earlier: $\frac{\text{change}}{\text{original number}}$

  What's the percentage increase if you go from 7 to 21?
  $\frac{14}{7} = 2 \times 100 = 200\%$

- A 100% decrease, on the other hand, leaves you at zero, no matter what.