Properties of real numbers Solutions NAME:
Distribution property and combining like terms

This worksheet will try to make the properties of real numbers more meaningful and memorable. We will use them a lot during the semester. Having them firmly in your head will make algebra easier. Particularly, we investigate the distribution property which is used crazy often. Combining like terms uses it even though it is not often pointed out.

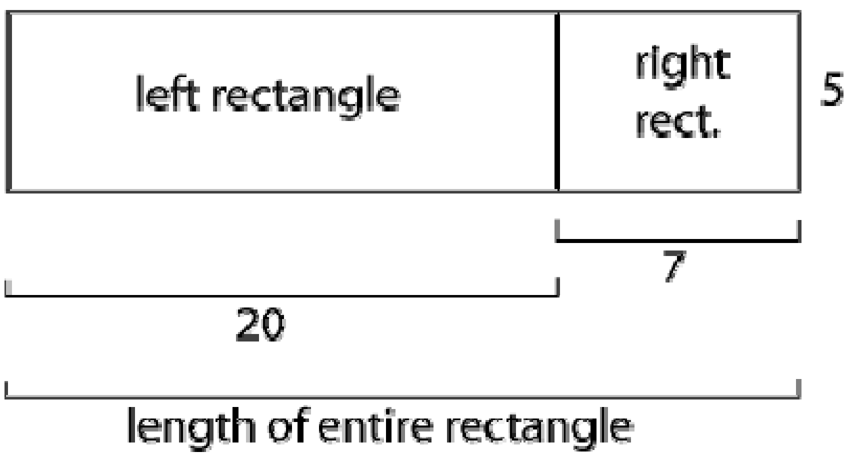
Distribution property $a*(b+c)=a*b+a*c$
To help understand how the distribution formula works, let's play with the rectangles below.

The area of a rectangle is its length times its width. For the three rectangles below, find

1.) the area of the left rectangle = ___*20*____ x ___*5*____ = ___*100*____,

2.) the area of the right rectangle = ____*7*__ x ___*5*___ = ____*35*_, and

3.) the area of the entire rectangle = ___*27*__ x __*5*____ = ___*135*_.



What do you notice about the answers from above? How can you justify what happens with the distribution property? Use the distribution property to write an equation concerning the areas found above.

The area of the left rectangle plus the area of the right rectangle equals the area of the entire rectangle. We see this by observing that 100 + 35 = 135. Also, we can see the distribution property here if we write this as $20*5+7*5=(20+7)*5$.

Let's work through some examples of the distribution property using actual numbers. For each side of the equation, work it out using the order of operations to simplify it. Notice this shows the equation is true. I'll work the first for you to show you what to do.

a.) $4*(3+6)=4*3+4*6$

left side: $4*(3+6)=4(9)=36$

Left: Do parentheses first.

right side: $4*3+4*6=12+24=36$

Right: Do multiplications first.

b.) $5*(10+3)=5*10+5*3$

left side: $5*13=65$

right side: $50+15=65$

c.) $2*(13-7)=2*13-2*7$

left side: $2*6=12$

right side: $26-14=12$

Notice the distribution property lets us write an expression two different ways:

1. as the sum of two terms with a common factor (right side above), or
2. as the product of two factors (left side above).

For the following sums, find the (largest) common factor between the two terms and use the distribution property to rewrite it as two factors. The first one is done for you. Notice the sum and the product you'll end up with are equivalent.

a.) 10 + 15

$= 5 * 2 + 5 * 3$

$= 5 * (2 + 3)$

$= 5 * 5$

Notice the two terms, 10 and 15, have a common factor of 5.

b.) 33 + 21

$= 3 * 11 + 3 * 7$

$= 3 * (11 + 7)$

$= 3 * 18$

$= 54$

c.) 36 – 54

$= 9(4) - 9(6)$

$= 9(4 - 6)$

$= 9(-2)$

$= -18$

Let's try it with a few variables. Remember, the variables represent real numbers, so by closure these expressions are just real numbers. We can factor and simplify algebraic expressions just like we do with real numbers. Notice in each example below, there is a common factor between the two terms. Factor it out with the distribution property.

a.) $3x^2 + 6y = 3(x^2 + 2y)$

b.) $4ab^2 + 2a = 2a(2b^2 + 1)$

c.) $5xy^2 - 20xyz = 5xy(y - 4z)$

Use what you have learned to simplify the following. Use the distribution property to factor the top, and then cancel common factors from top and bottom.

a.) $\dfrac{3x^2 + 6y}{3x} = \dfrac{3 * (x^2 + 2y)}{3 * x} = \dfrac{3}{3} * \dfrac{x^2 + 2y}{x} = \dfrac{x^2 + 2y}{x}$

The tops of these fractions are the same as the expressions we just worked with. If you get stuck, refer to how you simplified them before.

b.) $\dfrac{4ab^2 + 2a}{6a^2} = \dfrac{2a(2b^2 + 1)}{(2a)(3a)} = \dfrac{2a}{2a} * \dfrac{2b^2 + 1}{3a} = \dfrac{2b^2 + 1}{3a}$

c.) $\dfrac{5xy^2 - 20xyz}{y - 4z} = \dfrac{5xy(y - 4z)}{1 * (y - 4z)} = \dfrac{5xy}{1} * \dfrac{y - 4z}{y - 4z} = \dfrac{5xy}{1} = 5xy$

Any number divided by itself is 1. The expression "y – 4z" is just some number.

Combining like terms and the distribution property

Notice the terms of $3x^2 + 4x^2$ have a common factor of x^2. If we factor that out, we get $3x^2 + 4x^2 = (3+4)x^2 = 7x^2$. This is what we know as combining like terms but notice it is just the distribution property. Rewrite and simplify the following expressions. The first one is done for you.

a.) $6x + 4x$

$= (6+4)x$

$= 10x$

Factor out the *x*.

b.) $5a^2 + 7a^2$

$= (5+7)a^2$

$= 12a^2$

c.) $10y^2 - 5y^2$

$= (10-5)y^2$

$= 5y^2$

d.) $3x^2 + 8x^2 + 4y + 6y$

$= (3+8)x^2 + (4+6)y$

$= 11x^2 + 10y$

e.) $12xy + 4xy - 3x^2 + 7x^2$

$= (12+4)xy + (-3+7)x^2$

$= 16xy + 4x^2$