Properties of real numbers Solutions NAME:
Basics of numbers and algebra

This worksheet will try to make the properties of real numbers more meaningful and memorable. We will use them a lot during the semester. Having them firmly in your head will make algebra easier.

Definition of a real number
As you start out in algebra, you will likely only deal with real numbers. The real numbers are essentially every number you've seen so far in life except the imaginary (or complex numbers) such as 7 + 3*i* or $\sqrt{-5}$.

Real numbers include fractions (or rational numbers), zero, negatives, and even irrational numbers like $\sqrt{2}$ or π. Any number that describes something in your real life is essentially a real number.

Definition of an integer
An integer is a number in the set {…,-3, -2, -1, 0, 1, 2, 3,…}. We will refer to integers many times during the semester. We will also talk about **non-negative integers**; they are composed of the positive integers and zero.

Closure of real numbers over multiplication and addition
This property makes algebra work. It says if I take two real numbers and multiply, add, subtract, or divide them, I'll still have a real number when I'm through. This makes it possible to say that if *x* is a real number, then $\frac{4x^2+5x}{2x-3}$ is also a real number. The real numbers are said to be closed under addition, multiplication, and subtraction. (The real numbers are actually not closed under division. There is one real number that when we divide by it, you do **not** end up with another real number. Do you know which it is?)
This is important because as we deal with expressions like $\frac{4x^2+5x}{2x-3}$, we have to remember that all it is, is a real number.

We know a lot about real numbers and how they behave. To understand algebra, we have to somehow transfer that knowledge to algebraic expressions that represent real numbers.

This worksheet will help us investigate many properties of real numbers. We will explore a property using actual numbers, and then we look at how it is used with variables.

Factoring

Any real number can be written as a product of its factors. For instance, $45 = 5 * 9$. This allows us to reduce fractions such as $\frac{45}{10}$. We factor the top and bottom of the fraction, and cancel common factors: $\frac{45}{10} = \frac{5*9}{5*2} = \frac{5}{5} * \frac{9}{2} = \frac{9}{2}$. This allows us to mean exactly $\frac{45}{10}$, but write it more simply as $\frac{9}{2}$. Let's practice a couple before we move to algebra.

Simplify the fractions by factoring the top and bottom completely and canceling common factors like in the example above. Write it out explicitly like the above example so you internalize what is happening.

a.) $\frac{28}{48} = \frac{7*4}{12*4} = \frac{7}{12} * \frac{4}{4} = \frac{7}{12}$

b.) $\frac{60}{75} = \frac{15*4}{15*5} = \frac{15}{15} * \frac{4}{5} = \frac{4}{5}$

c.) Because expressions such as $4x^2y$ are real numbers, they are also factorable. What are the four factors of $4x^2y$? List them with commas.

The four factors are 4, x, x, and y. You can also think of it as three factors, 4, x^2, and y. The idea here is to start seeing expressions like this as a product of factors.

d.) Simplify the following algebraic expression. Notice the common factor of 4*xy* on top and bottom; factor both top and bottom and cancel the common factor. Write it out explicitly so you internalize what is happening.

$$\frac{4x^2y}{8xy^3} = \frac{4xy(x)}{4xy(2y^2)} = \frac{4xy}{4xy} * \frac{x}{2y^2} = \frac{x}{2y^2}$$

e.) Simplify the following algebraic expression. Notice the common factor of $7ab^2$ on top and bottom; factor both top and bottom and cancel the common factor. Write it out explicitly so you internalize what is happening.

$$\frac{35a^5b^2}{14ab^4} = \frac{7ab^2(5a^4)}{7ab^2(2b^2)} = \frac{7ab^2}{7ab^2} * \frac{5a^4}{2b^2} = \boxed{\frac{5a^4}{2b^2}}$$